

OCR

Oxford Cambridge and RSA

Friday 19 May 2017 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

- 1 Find $\sum_{r=1}^n (r^2 - r - 8)$, giving your answer in a fully factorised form. [5]
- 2 Use an algebraic method to find the square roots of the complex number $43 - (6\sqrt{10})i$. Give your answers in the form $x + iy$, where x and y are exact real numbers. [5]
- 3 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 1 & 4 \\ -2 & a \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 7 & 3 \\ 1 & 5 \end{pmatrix}$, where $a \neq -8$ and **I** is the 2×2 identity matrix. Find
- (i) $7\mathbf{A} - \mathbf{I}$, [2]
- (ii) $(\mathbf{A}^{-1}\mathbf{B}^{-1})^{-1}$. [3]
- 4 Prove by induction that, for $n \geq 1$, $\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$. [5]
- 5 The matrix $\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$ represents the transformation P.
- (i) Describe fully the transformation P. [3]
- Transformation Q is a stretch, parallel to the y-axis with scale factor 4.
- (ii) Find the matrix that represents transformation Q. [2]
- Transformation T is equivalent to transformation P followed by transformation Q.
- (iii) Find the matrix that represents transformation T. [2]
- (iv) Find the area of the image of the unit square under transformation T. [2]
- 6 The complex number z_1 has modulus 3 and argument $\frac{3}{5}\pi$. The complex number z_2 has modulus 3 and argument $-\frac{9}{10}\pi$.
- (i) Sketch on a single argand diagram z_1 , z_2 and $z_1 - z_2$. [3]
- (ii) Find the exact value of $|z_1 - z_2|$ and the exact value of $\arg(z_1 - z_2)$. [5]
- (iii) Give a geometrical description of the locus given by $|z - z_1| = |z - z_2|$. [2]

7 (i) Show that $\frac{1}{2r-1} - \frac{1}{2r+5} \equiv \frac{6}{(2r-1)(2r+5)}$. [1]

Hence find

(ii) $\sum_{r=2}^{30} \frac{6}{(2r-1)(2r+5)}$, giving your answer correct to 3 decimal places, [5]

(iii) $\sum_{r=2}^{\infty} \frac{6}{(2r-1)(2r+5)}$, giving your answer as a single fraction. [1]

8 In the cubic equation $4z^3 + az^2 + bz + c = 0$, a , b and c are real numbers. One root is $1 + \frac{3}{2}i$ and the sum of the roots is 6. Find the values of a , b and c . [7]

9 The matrix C is given by $C = \begin{pmatrix} a & 1 & 1 \\ 3 & a & 1 \\ 5 & 3 & 2 \end{pmatrix}$.

(i) Find the value of a for which C is singular. [5]

In the three simultaneous equations given below, p is a constant.

$$\begin{aligned} ax + y + z &= p \\ 3x + ay + z &= p - 1 \\ 5x + 3y + 2z &= p - 2 \end{aligned}$$

(ii) Write down one value of a for which these equations have a unique solution, giving a brief reason. [1]

(iii) Using the value of a found in (i), find the value of p for which these equations are consistent. [3]

10 The complex number $a + ib$ is denoted by z and the complex number $c + id$ is denoted by w .

It is given that $z^2 = z^*w$.

(i) Show that $2ab = ad - bc$. [4]

(ii) Given that the real part of $w = 0$, find the values of b in terms of a . [6]

END OF QUESTION PAPER

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